

Cavity enhanced rephased amplified spontaneous emission

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Abstract. Amplified spontaneous emission is usually treated as an incoherent noise process. Recent theoretical and experimental work using rephasing optical pulses has shown that rephased amplified spontaneous emission (RASE) is a potential source of wide bandwidth time-delayed entanglement. Due to poor echo efficiency the plain RASE protocol doesn't in theory achieve perfect entanglement. Experiments done to date show a very small amount of entanglement at best. Here we show that rephased amplified spontaneous emission can, in principle, produce perfect multimode time-delayed two mode squeezing when the active medium is placed inside a Q-switched cavity.

PACS numbers: 03.67.-a, 32.80.Qk, 42.50 p, 78.47.jf

1. Introduction

The bitrate available from current quantum networks falls off very quickly with increasing attenuation in the transmission path. Quantum repeaters [1] stand to alleviate this problem. The proposal of Duan-Lukin-Cirac and Zoller (DLCZ) [2] first suggested the use of atomic ensembles to generate time-separated entangled photons. There has been a lot of experimental progress in the use of atomic ensembles [3, 4, 5, 6, 7, 8, 9], but not yet a practical quantum repeater.

Quantum memories, like the DLCZ protocol, use the large interactions possible between an optical field and a collective excitation in an ensemble. There has been impressive demonstrations of quantum memories using photon echo techniques [10, 11, 12, 13, 14]. A distinct advantage such techniques have is that they are multimode [15, 16]. Ledingham et al. [17] suggested extending echo techniques to repeaters by using rephased amplified spontaneous emission (RASE) as a source of photon streams with time-separated entanglement. The RASE is illustrated in figure 1.

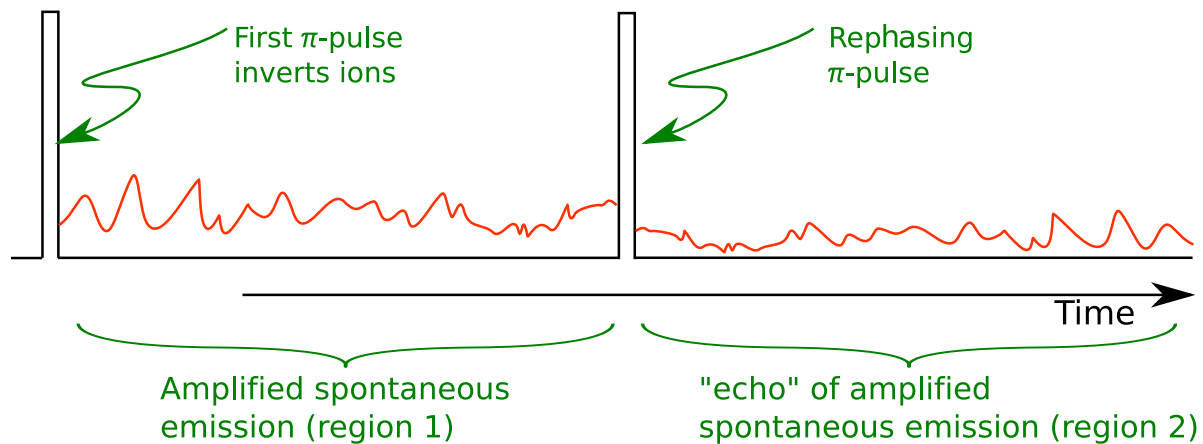


Figure 1. The RASE scheme. The first π -pulse shifts the two-level atoms into their excited state. Spontaneous emission from the excited atom ensemble produces the amplified spontaneous emission (ASE). During this emission, the atoms continually dephase due to inhomogeneous broadening. The rephasing π -pulse inverts the excitation of and rephases the atoms. Atoms that contributed to the ASE continue to emit photons to produce the RASE. This results in RASE that is an “echo” of the ASE. The RASE and ASE are correlated at times equally spaced from the rephasing π -pulse.

Realization of the RASE scheme has been demonstrated in two separate systems. One system follows the original proposal [18] and the other [19] implements a modified approach based on four level atoms [20]. Both demonstrations show that correlations exist between the ASE field and the RASE field. In the case of [18] the correlation was strong enough and the noise was low enough to show evidence of entanglement, although the confidence level was not high.

As a source of entanglement, RASE as it was initially proposed in [17] and [20] is

imperfect. The main problem is the efficiency of the recall of entanglement from the atomic ensemble. In the first step of [17] illustrated in figure 1, an inverted ensemble of atoms creates ASE. This light is entangled with collective degrees of freedom of the atomic ensemble. The excitation in those collective degrees of freedom is then recalled into an output light field with a rephasing π -pulse. The fact that the quality of entanglement was limited by this recall efficiency is problematic. The low optical depths desirable in the first step to get weak ASE lead to poor recall efficiency. The four-level RASE [20] is slightly better in that separate transitions are used for the ASE and RASE steps. This means that a weak transition can be used for the ASE step and a stronger one for the RASE. Recently [21] further improvement has been suggested by tailoring the spatial density of the ions.

The recall efficiency of the RASE scheme can be improved by placing the atoms inside a low finesse cavity, an approach also used in quantum memories [22, 15, 23]. In this paper we examine this process of cavity enhanced rephased amplified spontaneous emission (CRASE). We will show that in the appropriate regime the CRASE scheme is capable of achieving a recall efficiency of 100%, and in principle perfect multimode, time-separated entanglement.

2. The Hamiltonian for our system

The evolution of CRASE is qualitatively the same as the RASE evolution shown in figure 1. The interaction picture Hamiltonian for our system takes the form (making the rotating wave approximation and setting $\hbar \equiv 1$)

$$H = H_1 + H_2 \quad (1)$$

where

$$H_1 = \sum_{k=1}^N \sigma_+^k(t) \sigma_-^k(t) + ig \sum_{k=1}^N (\sigma_+^k(t) a(t) - \sigma_-^k(t) a^\dagger(t)) \quad (2)$$

is the usual Jaynes-Cummings Hamiltonian that models the interaction between N two levels atoms and the cavity mode and

$$H_2 = \int_{-\infty}^{\infty} \Delta b^\dagger(\Delta, t) b(\Delta, t) d\Delta + i \int_{-\infty}^{\infty} \kappa(\Delta) (b^\dagger(\Delta, t) a(t) - b(\Delta, t) a^\dagger(t)) d\Delta \quad (3)$$

models the interaction between the external radiation field and the cavity [24, 25]. The operators $\sigma_+^k(t)$ and $\sigma_-^k(t)$ are raising and lowering operators respectively for atom k , $a(t)$ is the destruction operator for the cavity mode, g is the coupling between the atoms and the cavity mode, which we take to be uniform, $b(\Delta, t)$ are destruction operators for the external radiation modes, Δ is the detuning from the cavity mode frequency and $\kappa(\Delta)$ is the coupling between the radiation mode with detuning Δ and the cavity mode.

3. The ASE field

At the beginning of region 1 in figure 1 the atoms are in their excited state. Spontaneous emission by the atoms produces the ASE field. We will assume that the atoms are weakly coupled to the cavity (small g) so that the atoms remain predominantly in their excited state throughout region 1. This allows us to approximate each atom as an *inverted* harmonic oscillator by setting $\sigma_+^k(t) \rightarrow s_k(t)$, where $s_k(t)$ are destruction operators satisfying $[s_k(t), s_{k'}^\dagger(t)] = \delta_{kk'}$ [24]. Like the ordinary harmonic oscillator, the eigenstates of an inverted harmonic oscillator form a ladder of equally spaced energy states. The ladder is inverted in the sense that an energy eigenstate $|n\rangle$ contains n units of *negative energy* so that the state $|n+1\rangle \propto s_k^\dagger |n\rangle$ has a lower energy than the state $|n\rangle$. We will also approximate the collection of harmonic oscillators by a continuous field by setting $\sqrt{N}s_k(t) \rightarrow s(\Delta, t)$. The operators $s(\Delta, t)$ are destruction operators for a collective excitation across oscillators with detuning Δ and satisfy $[s(\Delta, t), s^\dagger(\Delta', t)] = \delta(\Delta - \Delta')$ in the limit $N \rightarrow \infty$. The Hamiltonian H_1 then takes the form

$$H_1 = - \int_{-\infty}^{\infty} \Delta s^\dagger(\Delta, t) s(\Delta, t) d\Delta + i \int_{-\infty}^{\infty} g(\Delta) (s(\Delta, t) a(t) - s^\dagger(\Delta, t) a^\dagger(t)) d\Delta \quad (4)$$

where $\sqrt{N}g \rightarrow g(\Delta)$.

Input-output theory [24] is often used when describing quantum systems interacting with a continuum of radiation modes, such as the situation described by equation (3). The interaction between the excited state atoms and the cavity mode, described by equation (4), is very similar: in both cases the cavity is interacting with a continuum of harmonic oscillators. This allows us to use input-output theory for the atom-cavity interaction also.

Following the standard input-output treatment we assume $\kappa(\Delta)$ and $g(\Delta)$ are slowly varying for our range of frequencies of interest. We can then make the *first Markov approximation* by setting $\kappa(\Delta) \rightarrow \kappa(0) \equiv \sqrt{\gamma_{b,1}/2\pi}$ and $g(\Delta) \rightarrow g(0) \equiv \sqrt{\gamma_{a,1}/2\pi}$. In the case of g , this is valid when the inhomogeneous broadening of the atoms is much broader than the cavity bandwidth. The loss rate of the bare cavity is $\gamma_{b,1}$. We call $\gamma_{a,1}$ the ‘gain rate’ of the cavity; this is the rate describing the exponential growth of light in the cavity if the cavity mirrors were perfect. We will work in the regime where $\gamma_{a,1} < \gamma_{b,1}$ so that we are below the lasing threshold.

We are now able to obtain an expression for the output ASE field. Solving the Heisenberg equations of motion for $b(\Delta, t)$, $s(\Delta, t)$ and $a(t)$ gives

$$b(\Delta, t) = \exp[-i\Delta(t - t_0)] b(\Delta, t_0) + \sqrt{\frac{\gamma_{b,1}}{2\pi}} \int_{t_0}^t \exp[-i\Delta(t - \tau)] a(\tau) d\tau \quad (5)$$

$$s(\Delta, t) = \exp[i\Delta(t - t_0)] s(\Delta, t_0) - \sqrt{\frac{\gamma_{a,1}}{2\pi}} \int_{t_0}^t \exp[i\Delta(t - \tau)] a^\dagger(\tau) d\tau \quad (6)$$

and

$$a(t) = - \int_{t_0}^t \exp \left[-\frac{\gamma_{b,1} - \gamma_{a,1}}{2}(t - \tau) \right] \left(\sqrt{\gamma_{b,1}} b_{\text{in}}(\tau) + \sqrt{\gamma_{a,1}} s_{\text{in}}^\dagger(\tau) \right) d\tau \\ + \exp \left[-\frac{\gamma_{b,1} - \gamma_{a,1}}{2}(t - t_0) \right] a(t_0) \quad (7)$$

where

$$b_{\text{in}}(t) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp[-i\Delta(t - t_0)] b(\Delta, t_0) d\Delta \quad (8)$$

is the radiation field that enters the cavity at time t and

$$s_{\text{in}}(t) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp[i\Delta(t - t_0)] s(\Delta, t_0) d\Delta \quad (9)$$

is the input atomic field. Time $t_0 < 0$ occurs at the beginning of region 1 in figure 1. Both input fields are vacuum fields with $\langle b^\dagger(\Delta, t_0) b(\Delta', t_0) \rangle = \langle s^\dagger(\Delta, t_0) s(\Delta', t_0) \rangle = 0$. Note that we name the input atomic field from the point of view of the cavity mode; the input atomic field is the field that drives the cavity mode. Equations (5) and (6) can be used to derive the relations [24]

$$b_{\text{ASE}}(t) \equiv b_{\text{out}}(t) = b_{\text{in}}(t) + \sqrt{\gamma_{b,1}} a(t) \quad (10)$$

$$s_{\text{out}}(t) = s_{\text{in}}(t) - \sqrt{\gamma_{a,1}} a^\dagger(t) \quad (11)$$

where

$$b_{\text{out}}(t) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-i\Delta t) b(\Delta, 0) d\Delta \quad (12)$$

is the ASE field - the radiation field that exits the cavity at time t - and

$$s_{\text{out}}(t) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(i\Delta t) s(\Delta, 0) d\Delta \quad (13)$$

is the output atomic field. The output atomic field is a collective de-excitation of atoms, or equivalently an excitation of the inverted harmonic oscillator field.

We will assume that the frequencies of interest of the ASE field is narrowband compared to $\gamma_{b,1} - \gamma_{a,1}$, the net loss rate of the cavity. We can then adiabatically eliminate the cavity mode, replacing $b_{\text{in}}(\tau)$ by $b_{\text{in}}(t)$ and $s_{\text{in}}(\tau)$ by $s_{\text{in}}(t)$ in equation (7). Substituting the resulting expression for $a(t)$ into equations (10) and (11) and letting $t_0 \rightarrow -\infty$ gives the following input-output relations:

$$b_{\text{ASE}}(t) = -\frac{\gamma_{b,1} + \gamma_{a,1}}{\gamma_{b,1} - \gamma_{a,1}} b_{\text{in}}(t) - \frac{2\sqrt{\gamma_{b,1}\gamma_{a,1}}}{\gamma_{b,1} - \gamma_{a,1}} s_{\text{in}}^\dagger(t) \quad (14)$$

$$s_{\text{out}}(t) = \frac{\gamma_{b,1} + \gamma_{a,1}}{\gamma_{b,1} - \gamma_{a,1}} s_{\text{in}}(t) + \frac{2\sqrt{\gamma_{b,1}\gamma_{a,1}}}{\gamma_{b,1} - \gamma_{a,1}} b_{\text{in}}^\dagger(t) \quad (15)$$

These relations allow us to determine the output ASE field and output atomic field from the known input radiation and atomic fields.

4. The rephasing π -pulse

The rephasing π -pulse (applied at $t = 0$) induces the following changes in our atoms [26]:

$$\sigma_z^k(0) \rightarrow -\sigma_z^k(\delta t) \quad (16)$$

$$\sigma_+^k(0) \rightarrow \sigma_-^k(\delta t) \quad (17)$$

where δt is the duration of the π -pulse and $\sigma_z^k(t) = 2\sigma_+^k(t)\sigma_-^k(t) - I$, where I is the identity operator. The π -pulse inverts the excitation of and rephases the atoms. Like in [17], we model the rephasing pulse as an instantaneous π -pulse and so take $\delta t \rightarrow 0$. This is valid if $(\delta t)^{-1}$ is large compared to the bandwidth of the ASE and RASE fields that we are interested in.

5. The RASE field

After the rephasing π -pulse the atoms are predominantly in their ground state (equation (16)). But the atoms that fell to their ground state in region 1 will be in their excited state and these atoms produce the RASE field. Because the atoms are only weakly excited we approximate the atoms as a field of ordinary harmonic oscillators by setting $\sigma_-(\Delta, t) \rightarrow d(\Delta, t)$. Here $d(\Delta, t)$ are destruction operators satisfying $[d(\Delta, t), d^\dagger(\Delta', t)] = \delta(\Delta - \Delta')$.

The interaction picture Hamiltonian for our system takes the form (making the rotating wave approximation and setting $\hbar \equiv 1$)

$$\begin{aligned} H = & \int_{-\infty}^{\infty} \Delta b^\dagger(\Delta, t) b(\Delta, t) d\Delta + i\sqrt{\frac{\gamma_{b,2}}{2\pi}} \int_{-\infty}^{\infty} (b^\dagger(\Delta, t) a(t) - b(\Delta, t) a^\dagger(t)) d\Delta \\ & + \int_{-\infty}^{\infty} \Delta d^\dagger(\Delta, t) d(\Delta, t) d\Delta + i\sqrt{\frac{\gamma_{a,2}}{2\pi}} \int_{-\infty}^{\infty} (d^\dagger(\Delta, t) a(t) - d(\Delta, t) a^\dagger(t)) d\Delta \end{aligned} \quad (18)$$

The loss rate of the bare cavity is $\gamma_{b,1}$ and $\gamma_{a,1}$ is the rate the atoms would absorb photons from the cavity if the cavity mirrors were perfect. We have allowed for the atom-cavity and radiation-cavity couplings to differ from the couplings for times $t < 0$. This allows for the case where either the Q -factor of the cavity changes or the oscillator strength of the atomic transition used for the ASE and RASE are different [20].

We again make the first Markov approximation and assume that the frequencies of interest of the RASE field is narrowband compared to $\gamma_{a,1} + \gamma_{b,2}$, the total loss rate of the cavity, allowing us to again adiabatically eliminate the cavity mode. Carrying out analogous calculations to those from the previous section we obtain the following input-output relations:

$$b_{\text{RASE}}(t) \equiv b_{\text{out}}(t) = -\frac{\gamma_{b,2} - \gamma_{a,2}}{\gamma_{b,2} + \gamma_{a,2}} b_{\text{in}}(t) - \frac{2\sqrt{\gamma_{b,2}\gamma_{a,2}}}{\gamma_{b,2} + \gamma_{a,2}} d_{\text{in}}(t) \quad (19)$$

$$d_{\text{out}}(t) = \frac{\gamma_{b,2} - \gamma_{a,2}}{\gamma_{b,2} + \gamma_{a,2}} d_{\text{in}}(t) - \frac{2\sqrt{\gamma_{b,2}\gamma_{a,2}}}{\gamma_{b,2} + \gamma_{a,2}} b_{\text{in}}(t) \quad (20)$$

where

$$b_{\text{RASE}}(t) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp[-i\Delta(t - t_1)] b(\Delta, t_1) d\Delta \quad (21)$$

is the output RASE field, where t_1 is any time far in the future,

$$d_{\text{out}}(t) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp[-i\Delta(t - t_1)] d(\Delta, t_1) d\Delta \quad (22)$$

is the output atomic field,

$$d_{\text{in}}(t) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-i\Delta t) d(\Delta, 0) d\Delta \quad (23)$$

is the input atomic field and $b_{\text{in}}(t)$ is defined by equation (8).

Equation (19) gives the RASE field as a function of the known input radiation field and the currently unknown input atomic field. The input atomic field for region 2 can be related to the output atomic field from region 1 using equation (17) with $\sigma_+^k(0) \rightarrow s(\Delta, 0)$ and $\sigma_-^k(\delta t) \rightarrow d(\Delta, \delta t)$. The definitions of $d_{\text{in}}(t)$ and $s_{\text{out}}(t)$ (equations (23) and (13)) then give that

$$d_{\text{in}}(t) = s_{\text{out}}(-t) \quad (24)$$

in the limit $\delta t \rightarrow 0$. Equation (24) shows that the input atomic field that drives the cavity mode after the rephasing pulse is equal to the output atomic field that was driven by the cavity mode before the rephasing pulse. Entanglement between $s_{\text{out}}(t)$ and $b_{\text{ASE}}(t)$ before the rephasing pulse is translated into entanglement between $d_{\text{in}}(t)$ and $b_{\text{ASE}}(-t)$ as a result of equation (24). The field $b_{\text{RASE}}(t)$ becomes entangled with $d_{\text{in}}(t)$ and is therefore also entangled with $b_{\text{ASE}}(-t)$.

Using equations (24) and (15) in equation (19) gives

$$b_{\text{RASE}}(t) = -\frac{\gamma_{b,2} - \gamma_{a,2}}{\gamma_{b,2} + \gamma_{a,2}} b_{\text{in}}(t) - \frac{2\sqrt{\gamma_{b,2}\gamma_{a,2}}}{\gamma_{b,2} + \gamma_{a,2}} \left(\frac{\gamma_{b,1} + \gamma_{a,1}}{\gamma_{b,1} - \gamma_{a,1}} s_{\text{in}}(-t) + \frac{2\sqrt{\gamma_{b,1}\gamma_{a,1}}}{\gamma_{b,1} - \gamma_{a,1}} b_{\text{in}}^\dagger(-t) \right) \quad (25)$$

and this along with equation (14) give the RASE and ASE fields as functions of the input atomic field before the rephasing pulse and input radiation field. The input-output relations describing our system are shown graphically in figure 2.

6. Quantifying entanglement

To concentrate on a single temporal mode for both the ASE and RASE fields we introduce mode operators A and B defined by

$$A \equiv - \int_{-\infty}^0 g(t) b_{\text{ASE}}(t) dt \quad (26)$$

$$B \equiv \int_0^{\infty} g(t)^* b_{\text{RASE}}(t) dt \quad (27)$$

where $g(t)$ is a temporal mode function satisfying $g(-t) = g(t)$ and $\int_0^{\infty} |g(t)|^2 dt = 1$. The number of photons in the ASE and RASE modes are then given by $N_{\text{ASE}} = \langle A^\dagger A \rangle$ and $N_{\text{RASE}} = \langle B^\dagger B \rangle$

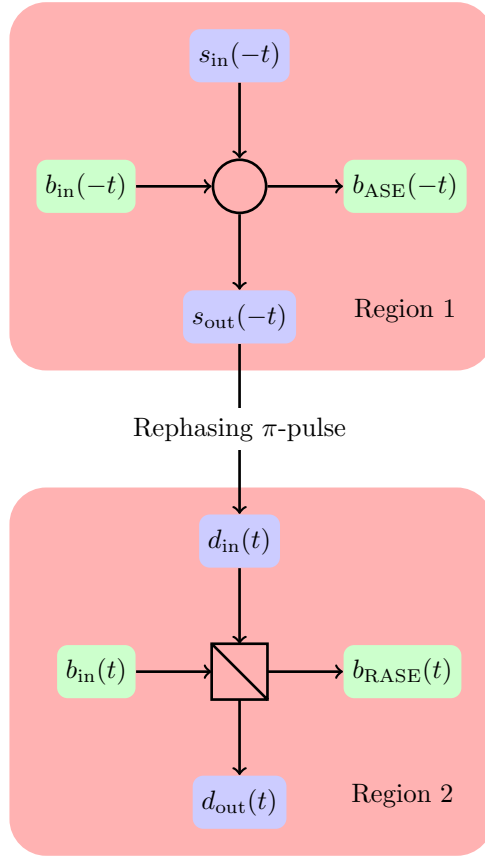


Figure 2. Graphical representation of the input-output relations (14) and (25). In the time region 1 the input optical field $b_{\text{in}}(-t)$ and the input atomic field $s_{\text{in}}(-t)$ combine like in a non-degenerate parametric amplifier to produce the outputs $b_{\text{ASE}}(-t)$ and $s_{\text{out}}(-t)$. The rephasing pulse means that the input atomic field for region 2 ($d_{\text{in}}(t)$) matches the output atomic field from region 1 ($s_{\text{out}}(-t)$). In region 2, the optical and atomic fields interact like on a beamsplitter. For an impedance matched cavity the reflectivity of this beamsplitter is 1. It is well known that in an impedance matched cavity full of atoms any input field gets totally absorbed by the atoms, and none of it escapes as light. What also happens in an impedance matched cavity is that the input atomic field is completely mapped onto the output optical field leading to 100% recall efficiency. This means that if the cavity is impedance matched in region 2, the two output optical fields $b_{\text{ASE}}(-t)$ and $b_{\text{RASE}}(t)$ will be maximally entangled.

Equations (14) and (25) give that

$$A = A_0 \cosh \chi + B_0^\dagger \sinh \chi \quad (28)$$

$$B = \sqrt{\epsilon} C_0 - \sqrt{1 - \epsilon} \left(B_0 \cosh \chi + A_0^\dagger \sinh \chi \right) \quad (29)$$

where $A_0 \equiv \int_{-\infty}^0 g(t) b_{\text{in}}(t) dt$, $B_0 \equiv \int_{-\infty}^0 g(t)^* s_{\text{in}}(t) dt$, $C_0 \equiv -\int_0^\infty g(t)^* b_{\text{in}}(t) dt$, $\cosh \chi \equiv (\gamma_{\text{b},1} + \gamma_{\text{a},1})/(\gamma_{\text{b},1} - \gamma_{\text{a},1})$ and $\sqrt{\epsilon} \equiv (\gamma_{\text{b},2} - \gamma_{\text{a},2})/(\gamma_{\text{b},2} + \gamma_{\text{a},2})$. We have that $[c_j, c_k^\dagger] = \delta_{jk}$ and $\langle c_j^\dagger c_k \rangle = 0$, with $c_j \in \{A_0, B_0, C_0\}$. Equations (28) and (29) take the form of the equations describing the output of a non-degenerate parametric amplifier and beam splitter combination [27], see figure 2.

Equations (28) and (29) give that $N_{\text{ASE}} = \sinh^2 \chi$ and $N_{\text{RASE}} = (1 - \epsilon) \sinh^2 \chi$. The recall efficiency of our system is given by $N_{\text{RASE}}/N_{\text{ASE}} = 1 - \epsilon$. If the cavity is impedance matched in region 2 ($\gamma_{a,2} = \gamma_{b,2}$) the recall efficiency is 100%, resulting in perfect entanglement between the ASE and RASE fields. Operation with the cavity impedance matched in region 2 requires either a Q -switched cavity or something equivalent to employing the 4 level scheme of [20] because, in order not to cross the lasing threshold, $\gamma_{b,1} > \gamma_{a,1}$ is required in region 1.

We will quantify the entanglement between the ASE and RASE fields using the criterion of Duan et al. [28]. For this criterion we introduce the operators

$$u \equiv \sqrt{\theta} x_A + \sqrt{1 - \theta} x_B \quad (30)$$

$$v \equiv \sqrt{\theta} p_A - \sqrt{1 - \theta} p_B \quad (31)$$

where $x_c \equiv (c + c^\dagger)/\sqrt{2}$ and $p_c \equiv -i(c - c^\dagger)/\sqrt{2}$ are amplitude and phase quadrature fields satisfying $[x_c, p_{c'}] = i\delta_{cc'}$, with $c, c' \in \{A, B\}$, and θ can be any real number in the interval $(0, 1)$. Then a sufficient condition for entanglement between the ASE and RASE photons is that $\langle \Delta u^2 \rangle + \langle \Delta v^2 \rangle < 1$ ‡. Figure 3 shows a contour plot of $\langle \Delta u^2 \rangle + \langle \Delta v^2 \rangle$, minimised with respect to θ , versus $\sqrt{\epsilon}$ and $\cosh \chi$. The ASE and RASE fields are entangled for all parameter values considered in this figure.

It can be shown that for our system

$$\langle \Delta u^2 \rangle + \langle \Delta v^2 \rangle = 1 + 2 \sinh^2 \chi - 2\epsilon(1 - \theta) \sinh^2 \chi - 4\sqrt{\theta - \theta^2} \sqrt{1 - \epsilon} \cosh \chi \sinh \chi \quad (32)$$

Setting $\theta = (1 - \epsilon)/(2 - \epsilon)$ gives $\langle \Delta u^2 \rangle + \langle \Delta v^2 \rangle < 1$ for all valid values of χ and ϵ . Therefore the ASE and RASE fields are entangled for all valid parameter values.

7. Implementation

The theoretical analysis of our system has made a number of simplifying assumptions and these will need to be considered carefully when it comes to implementation. It has been assumed that the rephasing π -pulse is a perfect π -pulse. The feasibility of applying a π -pulse to the whole ensemble is greatly helped by the area theorem [29], which states that a π -pulse will remain a π -pulse as it travels through resonant media. This has strong analogues in a cavity [30]. Of course, in practice the π -pulse won't be perfect. However, as discussed in [17], unwanted excitation of the atomic field will be temporally brief and will rapidly dephase and no longer interact with the cavity mode. The variation in driving strength of the atoms due to variations in the cavity mode field intensity could be removed using hole burning techniques [31, 32].

We have also ignored unwanted loss from our cavity. However, because of the moderate finesses required it should be easily feasible to reach a regime where the output coupling of the cavity is the dominant loss.

‡ The condition given in [28] is that the ASE and RASE photons are entangled if $\langle \Delta u^2 \rangle + \langle \Delta v^2 \rangle < \lambda^2 + 1/\lambda^2$, where $u \equiv |\lambda|x_A + (1/\lambda)x_B$ and $v \equiv |\lambda|p_A - (1/\lambda)p_B$ for some real, non-zero λ . Setting $\theta \equiv \lambda^2(\lambda^2 + 1/\lambda^2)^{-1}$ gives the condition used in this paper.

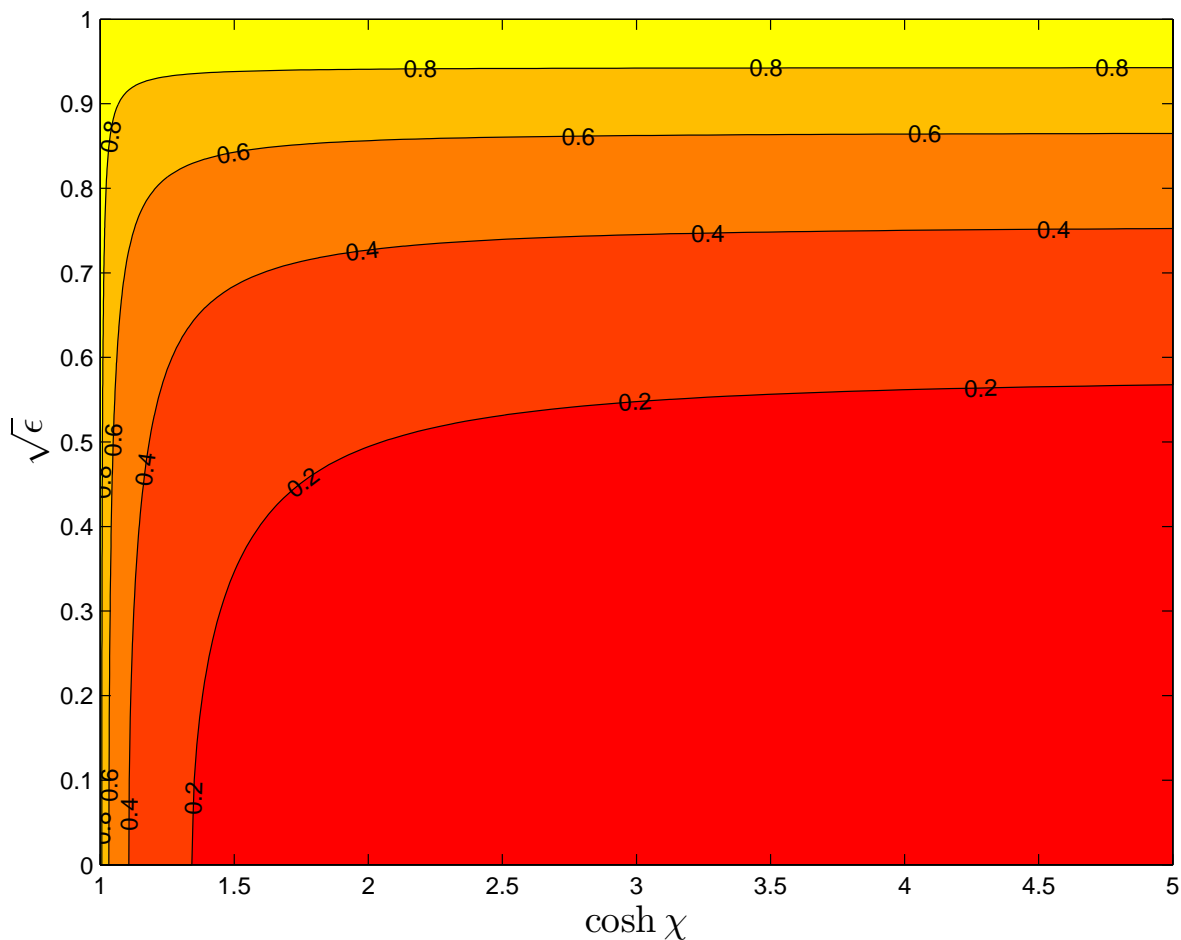


Figure 3. Contour plot of $\langle \Delta u^2 \rangle + \langle \Delta v^2 \rangle$, minimised with respect to θ , versus $\sqrt{\epsilon}$ and $\cosh \chi$. The ASE and RASE fields are necessarily entangled in regions where $\langle \Delta u^2 \rangle + \langle \Delta v^2 \rangle < 1$. The ASE and RASE fields are entangled for all parameter values considered in this figure.

8. Conclusion

In conclusion, we have analysed rephased amplified spontaneous emission with the atoms placed in an optical cavity. The cavity can alleviate the problem of low recall efficiency, particularly if the cavity is impedance matched when the entangled light is being recalled from the atoms, in which case the recall will theoretically be perfect. Achieving the impedance matched condition during recall requires either a Q -switched cavity or some way of switching the atoms' oscillator strength, such as in the 4 level scheme of Beavan et al. [20], since the ASE field has to be produced below the lasing threshold. We have also showed that entanglement exists between the ASE and RASE fields for all valid parameter values. Theoretically, our system has the potential to achieve time-separated entanglement with perfect recall efficiency, which is indispensable in producing an effective quantum repeater.

Acknowledgements

The authors would like to acknowledge financial support from the Marsden Fund of the Royal Society of New Zealand.

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